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Conformal field theory for the superstring in a Ramond-Ramond plane wave background

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Abstract: A quantizable worldsheet action is constructed for the superstring in a supersymmetric plane wave background with Ramond-Ramond flux. The action is manifestly invariant under all isometries of the background and is an exact worldsheet conformal field theory.

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1. Introduction

It has recently been recognized that the Penrose limit of the $AdS_5 \times S^5$ background with Ramond-Ramond (R-R) flux is a supersymmetric plane wave [1], and the superstring in this background is described in the light-cone Green-Schwarz (GS) formalism [2] by a quadratic worldsheet action [3, 4]. The spectrum of physical states with nonzero P^+ momentum can easily be computed using this light-cone GS action, which has been useful for checking aspects of the Maldacena conjecture [5]. However, to compute tree amplitudes or to describe physical states with vanishing P^+ momentum, the light-cone GS formalism is problematic even in a flat background. As will be discussed below, the problems associated with light-cone gauge become even more troublesome in the plane wave background since there is no J^{+-} isometry. Furthermore, it would be convenient to have a quantizable worldsheet action in which all isometries of the plane wave background are manifest, and not just those isometries which commute with the light-cone gauge choice.

Although the covariant GS action [6, 3] can classically describe the plane wave background in a manner which preserves all isometries, it is not known how to covariantly quantize the GS action. Over the last eight years, an alternative formalism for the superstring has been developed which can be covariantly quantized [7, 8, 9]. In a flat background, this formalism has a quadratic worldsheet action and tree amplitudes can be easily computed in a manifestly super-Poincaré covariant manner. The formalism generalizes to curved backgrounds [10] and was used to construct quantizable actions for the superstring in $AdS_5 \times S^5$ [9, 11], $AdS_3 \times S^3$ [12, 13] and $AdS_2 \times S^2$ [14] backgrounds with R-R flux.

In this paper, this alternative formalism will be used to construct a quantizable action for the superstring in the plane wave background obtained by taking the Penrose limit of these $AdS_{D/2} \times S^{D/2}$ backgrounds. By using the "pure spinor" or "hybrid" versions of the formalism, all isometries of the plane wave background can be made manifest. The action is not quadratic, which is not surprising since the action for the bosonic plane wave background is not quadratic when written in conformal gauge. However, the action is simpler than its $AdS_{D/2} \times S^{D/2}$ counterpart and can be proven to be an exact conformal field theory.

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In section 2 of this paper, the limitations of the light-cone GS formalism will be discussed. In section 3, a covariantly quantizable action will be constructed for the Penrose limit of the $AdS_5 \times S^5$, $AdS_3 \times S^3$, and $AdS_2 \times S^2$ backgrounds with R-R flux. And in section 4, the action will be proven to be an exact conformal field theory.

2. Limitations of the light-cone GS formalism

Since the light-cone action only depends on physical worldsheet variables, the light-cone formalism for the bosonic string and superstring is the most efficient way to compute the physical spectrum at non-zero P^+ momentum. However, besides the lack of manifest covariance, there are various other drawbacks of the light-cone GS action which are not present in the covariant action.[1]

One drawback is that the light-cone gauge is only well-defined when P^+ is nonzero, so the light-cone action cannot be used to obtain the spectrum at vanishing P^+ momentum. Although in a flat background, one can always rotate any state with nonzero momentum to have nonzero P^+ momentum, this is not always possible in backgrounds such as the plane wave background where SO(9, 1) covariance is absent. So the light-cone formalism in a plane wave background may be unable to describe certain non-trivial physical states.

Another drawback of the light-cone formalism which is especially problematic for the light-cone GS formalism is the explicit dependence on interaction points in the computation of scattering amplitudes. Recall that N-point tree amplitudes in light-cone gauge are computed using the Mandelstam map [15]

$$\rho(z) = \sum_{r=1}^{N} P_r^+ \log(z - z_r)\,, \tag{2.1}$$

where $\rho(z)$ maps the complex plane into the interacting string diagram and P_r^+ is the P^+ momentum of the r^{th} external string. For the bosonic string in light-cone gauge, interactions are described by a simple overlap integral, so scattering amplitudes can be easily computed by evaluating correlation functions of light-cone vertex operators located at $z = z_r$ in the complex plane, which get mapped to $\rho = \pm\infty$ in the string diagram.

However, for the GS superstring in light-cone gauge, interactions are not simply overlap integrals but also include an explicit operator which must be inserted at the interaction point [16, 17].[2] Using SU(4) × U(1) notation, this interaction point operator is

$$\left| \partial x_L + \partial x_{[\mu\nu]} S^\mu S^\nu + \frac{1}{24} \partial x_R \epsilon_{\mu\nu\rho\sigma} S^\mu S^\nu S^\rho S^\sigma \right|^2 , \tag{2.2}$$

where $\mu = 1$ to 4 is an SU(4) index, $[S^\mu, S_\mu]$ and $[\overline{S}^\nu, \overline{S}_\nu]$ are the left and right-moving SO(8) spinors decomposed in terms of a $[4_{\frac{1}{2}}, \overline{4}_{-\frac{1}{2}}]$ representation of SU(4) × U(1), and

[1] This section is based on several discussions with Michael Green.

[2] For the RNS superstring in light-cone gauge, one also needs to include an interaction point operator when using the Mandelstam map of (2.1) to describe the string diagram [18]. However, if one instead describes the string diagram using the map $\rho(z, \theta) = \sum_{r=1}^{N} P_r^+ \log(z - z_r - \theta\theta_r)$ where (z, θ) parameterizes a complex "super-plane", one can avoid interaction point operators in the light-cone RNS formalism [19].

$[x_L, x_{[\mu\nu]}, x_R]$ is the SO(8) vector decomposed in terms of a $[1_1, 6_0, 1_{-1}]$ representation of $\mathrm{SU}(4) \times \mathrm{U}(1)$. For an N-point tree amplitude described by the map of (2.1), the $N-2$ interaction point operators are located at the points z_κ where

$$\left.\frac{\partial \rho}{\partial z}\right|_{z=z_\kappa} = \sum_{r=1}^{N} \frac{P_r^+}{z_\kappa - z_r} = 0\,. \tag{2.3}$$

So scattering amplitudes are computed by evaluating correlation functions which involve both light-cone vertex operators at $z = z_r$ and interaction point operators at $z = z_\kappa$. Since expressing z_κ in terms of z_r requires finding the zeros of a polynomial of degree $N-2$, the light-cone GS formalism has not yet been used to compute tree amplitudes with more than four external strings.[3] Furthermore, singularities occuring when interaction points collide imply that one needs to include contact terms in the light cone interaction to remove these singularities [21] . The precise form of these light-cone contact terms has not been worked out. Note that in a covariant formalism, these problems associated with light-cone interaction point operators are not present since one can always "smooth out" the interaction point using a conformal transformation.

A third drawback of the light-cone formalism is that in backgrounds which are not invariant under the J^{+-} Lorentz transformation, the light-cone action is complicated when written in the complex plane. For example, in the supersymmetric plane wave background, the light-cone GS action is [3]

$$\mathcal{S} = \int d^2\rho \left(\frac{1}{2}\partial_\rho x^j \overline{\partial}_{\overline{\rho}} x^j + S^a \overline{\partial}_{\overline{\rho}} S^a + \overline{S}^a \partial_\rho \overline{S}^a + \frac{1}{2}\mu^2 x^j x^j + 2\mu S^a \sigma^{1234}_{ab} \overline{S}^b \right), \tag{2.4}$$

where ρ parameterizes the interacting string diagram, $a = 1$ to 8 is an SO(8) spinor index, and $F^{-1234} = F^{-5678} = \mu$ is the R-R flux.

Using the Mandelstam map of (2.1), the action in the complex plane is therefore

$$\begin{aligned}\mathcal{S} = \int d^2 z \Bigg(& \frac{1}{2}\partial_z x^j \overline{\partial}_{\overline{z}} x^j + S^a_z \overline{\partial}_{\overline{z}} S^a_z + \overline{S}^a_{\overline{z}} \partial_z \overline{S}^a_{\overline{z}} + \\ & + \frac{\mu^2}{2} \left| \sum_{r=1}^{N} \frac{P_r^+}{z - z_r} \right|^2 x^j x^j + 2\mu \left| \sum_{r=1}^{N} \frac{P_r^+}{z - z_r} \right| S^a_z \sigma^{1234}_{ab} \overline{S}^b_{\overline{z}} \Bigg),\end{aligned} \tag{2.5}$$

where $S^a_z = (\partial\rho/\partial z)^{\frac{1}{2}} S^a$ and $\overline{S}^a_{\overline{z}} = (\partial\overline{\rho}/\partial\overline{z})^{\frac{1}{2}} \overline{S}^a$.[4] This means that the Green's function $G^{jk}(z, z')$ for $\langle x^j(z) z^k(z')\rangle$ in the complex plane must satisfy the complicated differential

[3] One trick [20] for computing light-cone GS amplitudes in a flat background is to choose a Lorentz frame in which $P_r^+ \to 0$ for $r = 2$ to $N-1$. In this "short string" limit, $z_\kappa \to z_r$ for $r = 2$ to $N-1$ and the interaction point operator combines with the light-cone vertex operator to give an operator which resembles the covariant vertex operator in light-cone gauge. After computing the scattering amplitude in this "short string" limit, one can then use SO(9,1) covariance to derive the amplitude for generic values of P_r^+. However, this trick does not work in the plane wave background because of the absence of SO(9,1) covariance.

[4] It is interesting to note that the μ dependence of the action of (2.5) drops out near the light-cone interaction points z_κ satisfying (2.3). This suggests that the light-cone interaction point operator in a plane wave background is the same as the light-cone interaction point operator of (2.2) in a flat background. I would like to thank Michael Green for discussions on this point.

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equation

$$\left(\partial_z\overline{\partial}_{\overline{z}} - \mu^2\left|\sum_{r=1}^{N}\frac{P_r^+}{z-z_r}\right|^2\right)G^{jk}(z,z') = \eta^{jk}\delta^2(z-z')\,. \tag{2.6}$$

Finding a solution to (2.6) is probably no easier than computing OPE's using a conformally invariant action which is not quadratic.

So although the quadratic action in the light-cone GS formalism is extremely useful for computing the physical spectrum at nonzero P^+ momentum, it is not convenient for describing physical states with vanishing P^+ momentum or for computing tree-level scattering amplitudes.

3. Covariant action for the superstring in R-R plane wave background

In this section, a quantizable action will be constructed for the supersymmetic plane wave background coming from the Penrose limit of the $AdS_5\times S^5$, $AdS_3\times S^3$ and $AdS_2\times S^2$ backgrounds with R-R flux. Although the action will have features in common with the covariant GS action in these backgrounds, there are some important differences which allow covariant quantization.

One difference is the presence of the worldsheet variables d_α and $\overline{d}_{\overline{\alpha}}$ which play the role of conjugate momenta to the left and right-moving θ^α and $\overline{\theta}^{\overline{\alpha}}$ variables.[5] These conjugate momentum variables break kappa symmetry, which is replaced by BRST invariance in the $D=10$ version of the action and by $N=2$ worldsheet superconformal invariance in the $D=6$ and $D=4$ versions of the action. A second difference with the covariant GS formalism is the presence of bosonic worldsheet ghost variables. In the $D=10$ version of the action, these worldsheet ghosts transform as pure spinors under Lorentz transformations, while in the $D=6$ and $D=4$ versions, they are Lorentz scalars.

Since the alternative formalism for the superstring can be defined in any consistent supergravity background, it is straightforward to construct the worldsheet action in any specific background. In either the $AdS_{D/2}\times S^{D/2}$ background with R-R flux or its corresponding plane wave limit, the worldsheet action is

$$\mathcal{S} = \mathcal{S}_{GS} + \int d^2z\left(d_\alpha\overline{L}^\alpha + \overline{d}_{\overline{\alpha}}L^{\overline{\alpha}} - \frac{1}{2}d_\alpha\overline{d}_{\overline{\beta}}F^{\alpha\overline{\beta}}\right) + \mathcal{S}_{\rm comp} + \mathcal{S}_{\rm ghost} \tag{3.1}$$

where $F^{m_1\cdots m_{\frac{D}{2}}}$ is the constant $D/2$-form self-dual Ramond-Ramond flux and $F^{\alpha\overline{\beta}} = \frac{1}{(D/2)!}F^{m_1\cdots m_{\frac{D}{2}}}(\gamma_{m_1\cdots m_{D/2}})^{\alpha\overline{\beta}}$.

The first term $\mathcal{S}_{GS}$ in (3.1) is the standard covariant GS action

$$\mathcal{S}_{GS} = \int d^2z\left[\frac{1}{2}\eta_{mn}L^m\overline{L}^n + \int dy\,\epsilon^{IJK}\left(\gamma_{m\alpha\beta}L_I^mL_J^\alpha L_K^\beta + \gamma_{m\overline{\alpha}\overline{\beta}}L_I^mL_J^{\overline{\alpha}}L_K^{\overline{\beta}}\right)\right] \tag{3.2}$$

[5] Even though θ^α and $\overline{\theta}^{\overline{\alpha}}$ have the same chirality for the Type IIB superstring, it will be convenient to distinguish the spinor indices on these left and right-moving variables by using barred or unbarred indices.

constructed using the Metsaev-Tseytlin currents [22, 3]

$$
\begin{aligned}
G^{-1}\partial G &= P_m L^m + Q_\alpha L^\alpha + Q_{\overline{\alpha}} L^{\overline{\alpha}} + \frac{1}{2} J_{mn} L^{mn}\,, \\
G^{-1}\overline{\partial} G &= P_m \overline{L}^m + Q_\alpha \overline{L}^\alpha + Q_{\overline{\alpha}} \overline{L}^{\overline{\alpha}} + \frac{1}{2} J_{mn} \overline{L}^{mn}\,,
\end{aligned}
\tag{3.3}
$$

where $G(x^m, \theta^\alpha, \overline{\theta}^{\overline{\alpha}}) = \exp(x^m P_m + \theta^\alpha Q_\alpha + \overline{\theta}^{\overline{\alpha}} Q_{\overline{\alpha}})$, $[x^m, \theta^\alpha, \overline{\theta}^{\overline{\alpha}}]$ are $N = 2$ D-dimensional superspace variables[6] with $m = 0$ to $D-1$ and $[\alpha, \overline{\alpha}] = 1$ to $(2D-4)$, the generators $[P_m, Q_\alpha, Q_{\overline{\alpha}}, J_{mn}]$ form a super-Lie algebra with the commutation relations

$$
\begin{aligned}
&[P^m, P^n] = \frac{1}{2} R^{mnpq} J_{pq}\,, \qquad \{Q_\alpha, Q_\beta\} = 2\gamma^m_{\alpha\beta} P_m\,, \qquad \{Q_{\overline{\alpha}}, Q_{\overline{\beta}}\} = 2\gamma^m_{\overline{\alpha}\overline{\beta}} P_m\,, \\
&[Q_\alpha, P^m] = \gamma^m_{\alpha\beta} F^{\beta\overline{\gamma}} Q_{\overline{\gamma}}\,, \qquad [Q_{\overline{\alpha}}, P^m] = -\gamma^m_{\overline{\alpha}\overline{\beta}} F^{\gamma\overline{\beta}} Q_\gamma\,, \qquad \{Q_\alpha, Q_{\overline{\gamma}}\} = \frac{1}{2} J_{[mn]} \gamma^m_{\alpha\beta} F^{\beta\overline{\delta}} \gamma^n_{\overline{\delta}\overline{\gamma}}\,,
\end{aligned}
\tag{3.4}
$$

J_{mn} generate the usual Lorentz algebra, R^{mnpq} is the spacetime curvature which is related to $F^{\alpha\overline{\beta}}$ by the identity

$$
R^{mnpq} (\gamma_{pq})_\alpha^{\ \beta} = \gamma^m_{\alpha\gamma} F^{\gamma\overline{\delta}} \gamma^n_{\overline{\delta}\overline{\kappa}} F^{\beta\overline{\kappa}} - \gamma^n_{\alpha\gamma} F^{\gamma\overline{\delta}} \gamma^m_{\overline{\delta}\overline{\kappa}} F^{\beta\overline{\kappa}}\,,
\tag{3.5}
$$

$\gamma^m_{\alpha\beta}$ and $\gamma^m_{\overline{\alpha}\overline{\beta}}$ are $(2D-4) \times (2D-4)$ symmetric γ-matrices, and $\int dy \epsilon^{IJK} (\gamma_{m\alpha\beta} L^m_I L^\alpha_J L^\beta_K + \gamma_{m\overline{\alpha}\overline{\beta}} L^m_I L^{\overline{\alpha}}_J L^{\overline{\beta}}_K)$ is the Wess-Zumino term which is constructed such that $\mathcal{S}_{GS}$ is invariant under κ-symmetry. Under $G \to \Omega G H$ for global Ω and local H, the currents $G^{-1}\partial G$ are invariant up to a tangent-space Lorentz rotation using the standard coset construction where $[P_m, Q_\alpha, Q_{\overline{\alpha}}, J_{mn}]$ are the generators in Ω and J_{mn} are the generators in H. So as long as the action is constructed from Lorentz-invariant combinations of currents, the action is invariant under the global target-space isometries generated by $[P_m, Q_\alpha, Q_{\overline{\alpha}}, J_{mn}]$. Note that because the R-R field-strength is self-dual, only $(3D-10)$ of the $\frac{1}{2}D(D-1)$ Lorentz generators J_{mn} appear in (3.4). So only $(3D-10)$ of the L^{mn} currents are nonzero in (3.3).

The terms $d_\alpha \overline{L}^\alpha$ and $\overline{d}_{\overline{\alpha}} L^{\overline{\alpha}}$ in (3.1) break kappa symmetry but allow quantization since they imply non-vanishing propagators for θ^α and $\overline{\theta}^{\overline{\alpha}}$. And the term $-\frac{1}{2} d_\alpha \overline{d}_{\overline{\beta}} F^{\alpha\overline{\beta}}$ comes from the R-R vertex operator and implies that certain components of d_α and $\overline{d}_{\overline{\beta}}$ are auxiliary fields. For the $D = 10$ background, the term $\mathcal{S}_{\text{comp}}$ is absent, while for the $D = 6$ and $D = 4$ backgrounds, $\mathcal{S}_{\text{comp}}$ is the action for an $N = 2$ $c = \frac{3}{2}(10 - D)$ superconformal field theory which describes the $(10-D)$-dimensional compactification manifold.

Finally, $\mathcal{S}_{\text{ghost}}$ describes the action for the worldsheet ghosts. This action is non-trivial in the $D = 10$ background since the $D = 10$ ghosts transform under Lorentz transformations and therefore couple through their Lorentz currents to the spacetime connection and curvature of the background. In $D = 10$, this ghost action is

$$
\mathcal{S}_{\text{ghost}} = \int d^2 z \left[\mathcal{L}^{\text{flat}}_{\text{ghost}} + \frac{1}{2} N_{mn} \overline{L}^{mn} + \frac{1}{2} \overline{N}_{mn} L^{mn} + \frac{1}{4} N_{mn} \overline{N}_{pq} R^{mnpq} \right],
\tag{3.6}
$$

[6] In $D = 6$, the action of (3.1) uses the hybrid superstring formalism with eight θ's and eight $\overline{\theta}$'s. As discussed in [13], the $D = 6$ action of [12] using the hybrid superstring formalism with four θ's and four $\overline{\theta}$'s can be obtained from (3.1) by using the "harmonic" constraint to gauge away $(\theta^{\alpha 2}, \overline{\theta}^{\overline{\alpha} 2})$ and to replace $(d_{\alpha 2}, \overline{d}_{\overline{\alpha} 2})$ with $(d_{\alpha 1} e^{-\rho - i\sigma}, \overline{d}_{\overline{\alpha} 1} e^{-\overline{\rho} - i\overline{\sigma}})$.

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where $\mathcal{L}^{\rm flat}_{\rm ghost}$ is the free lagrangian in a flat background for the left and right-moving worldsheet ghosts $(\lambda^\alpha, w_\alpha)$ and $(\overline{\lambda}^{\overline{\alpha}}, \overline{w}_{\overline{\alpha}})$, λ^α and $\overline{\lambda}^{\overline{\alpha}}$ are pure spinors satisfying $\lambda\gamma^m\lambda = \overline{\lambda}\gamma^m\overline{\lambda} = 0$ for $m = 0$ to 9, w_α and $\overline{w}_{\overline{\alpha}}$ are their conjugate momenta, $N_{mn} = \frac{1}{2}\lambda\gamma_{mn}w$ and $\overline{N}_{mn} = \frac{1}{2}\overline{\lambda}\gamma_{mn}\overline{w}$ are their left and right-moving Lorentz currents, and R^{mnpq} is the target-space curvature tensor. Note that $\mathcal{S}_{\rm ghost}$ is invariant under local tangent-space Lorentz rotations, which is necessary for the action to be well-defined on the coset superspace described by $G(x,\theta,\overline{\theta})$. In the $D = 6$ and $D = 4$ actions, the worldsheet ghosts are Lorentz scalars so their Lorentz currents vanish and $\mathcal{S}_{\rm ghost} = \mathcal{S}^{\rm flat}_{\rm ghost}$.

The various terms in (3.1) have been chosen such that the $D = 10$ action is BRST invariant and such that the $D = 6$ and $D = 4$ actions are $N = 2$ worldsheet superconformally invariant. To check these invariances at the classical level, it is useful to compute the equations of motion for d_α and $\overline{d}_{\overline{\alpha}}$.

Suppose one varies $Z^M = [x^m, \theta^\alpha, \overline{\theta}^{\overline{\alpha}}]$ such that $E^\alpha_M \delta Z^M = \rho^\alpha$, $E^{\overline{\alpha}}_M \delta Z^M = \overline{\rho}^{\overline{\alpha}}$, and $E^m_M \delta Z^M = 0$ where $L^\alpha = E^\alpha_M \partial Z^M$, $L^{\overline{\alpha}} = E^{\overline{\alpha}}_M \partial Z^M$, $L^m = E^m_M \partial Z^M$, and $[L^\alpha, L^{\overline{\alpha}}, L^m]$ are defined in (3.3). Then the covariant GS action $\mathcal{S}_{GS}$ transforms as

$$\delta\mathcal{S}_{GS} = 2\rho^\alpha L^m \gamma_{m\alpha\beta}\overline{L}^\beta + 2\overline{\rho}^{\overline{\alpha}}\overline{L}^m\gamma_{m\overline{\alpha}\overline{\beta}}L^{\overline{\beta}}\,. \tag{3.7}$$

The transformation of (3.7) is related to kappa symmetry since when $\rho^\alpha = \kappa_\beta L^m \gamma^{\alpha\beta}_m$ and $\overline{\rho}^{\overline{\alpha}} = \overline{\kappa}_{\overline{\beta}} L^m \gamma^{\overline{\alpha}\overline{\beta}}_m$, $\delta\mathcal{S}_{GS}$ is proportional to the Virasoro constraints $\eta_{mn}L^m L^n$ and $\eta_{mn}\overline{L}^m\overline{L}^n$.

Furthermore, the commutation relations of (3.4) imply that

$$\begin{aligned}
\delta L^\alpha &= \partial\rho^\alpha + \frac{1}{4}(\gamma^{mn})^\alpha_\beta L_{mn}\rho^\beta + F^{\alpha\overline{\beta}}\gamma^m_{\overline{\beta}\overline{\gamma}}L_m\overline{\rho}^{\overline{\gamma}}\,,\\
\delta L^{\overline{\alpha}} &= \partial\overline{\rho}^{\overline{\alpha}} + \frac{1}{4}(\gamma^{mn})^{\overline{\alpha}}_{\overline{\beta}} L_{mn}\overline{\rho}^{\overline{\beta}} - F^{\beta\overline{\alpha}}\gamma^m_{\beta\gamma}L_m\rho^\gamma\,,\\
\delta L^{mn} &= (\gamma^{[m}F\gamma^{n]})_{\beta\overline{\gamma}}\rho^\beta L^{\overline{\gamma}} + (\gamma^{[m}F\gamma^{n]})_{\beta\overline{\gamma}}L^\beta\overline{\rho}^{\overline{\gamma}}\,,
\end{aligned} \tag{3.8}$$

where $(\gamma^{[m}F\gamma^{n]})_{\alpha\overline{\delta}} = \frac{1}{2}(\gamma^m_{\alpha\beta}F^{\beta\overline{\gamma}}\gamma^n_{\overline{\gamma}\overline{\delta}} - \gamma^n_{\alpha\beta}F^{\beta\overline{\gamma}}\gamma^m_{\overline{\gamma}\overline{\delta}})$.

So by varying ρ^α and $\overline{\rho}^{\overline{\alpha}}$, one obtains the equations of motion

$$\begin{aligned}
\overline{\partial}d_\alpha &= 2\gamma^m_{\alpha\beta}L_m\overline{L}^\beta + \frac{1}{4}d_\beta(\gamma_{mn})^\beta_\alpha\overline{L}^{mn} - \overline{d}_{\overline{\beta}}F^{\gamma\overline{\beta}}\gamma^m_{\gamma\alpha}L_m + \frac{1}{2}(\gamma^{[m}F\gamma^{n]})_{\alpha\overline{\gamma}}(N_{mn}\overline{L}^{\overline{\gamma}} + \overline{N}_{mn}L^{\overline{\gamma}})\,,\\
\partial\overline{d}_{\overline{\alpha}} &= 2\gamma^m_{\overline{\alpha}\overline{\beta}}\overline{L}_m L^{\overline{\beta}} + \frac{1}{4}\overline{d}_{\overline{\beta}}(\gamma_{mn})^{\overline{\beta}}_{\overline{\alpha}}L^{mn} + d_\beta F^{\beta\overline{\gamma}}\gamma^m_{\overline{\gamma}\overline{\alpha}}\overline{L}_m - \frac{1}{2}(\gamma^{[m}F\gamma^{n]})_{\gamma\overline{\alpha}}(N_{mn}\overline{L}^\gamma + \overline{N}_{mn}L^\gamma)\,.
\end{aligned} \tag{3.9}$$

Plugging into (3.9) the equations of motion $\overline{L}^\alpha = \frac{1}{2}F^{\alpha\overline{\beta}}\overline{d}_{\overline{\beta}}$ and $L^{\overline{\alpha}} = -\frac{1}{2}F^{\beta\overline{\alpha}}d_\beta$ which come from varying d_α and $\overline{d}_{\overline{\alpha}}$, one finds

$$\begin{aligned}
\overline{\nabla}d_\alpha &= \frac{1}{2}(\gamma^{[m}F\gamma^{n]})_{\alpha\overline{\gamma}}\left(N_{mn}\overline{L}^{\overline{\gamma}} - \frac{1}{2}\overline{N}_{mn}F^{\delta\overline{\gamma}}d_\delta\right)\,,\\
\nabla\overline{d}_{\overline{\alpha}} &= -\frac{1}{2}(\gamma^{[m}F\gamma^{n]})_{\gamma\overline{\alpha}}\left(\frac{1}{2}N_{mn}F^{\gamma\overline{\delta}}\overline{d}_{\overline{\delta}} + \overline{N}_{mn}L^\gamma\right)\,,
\end{aligned} \tag{3.10}$$

where the spin connections in the covariantized derivatives ∇ and $\overline{\nabla}$ are L^{mn} and $\overline{L}^{mn}$.

When $D=10$, BRST invariance implies that the left and right-moving BRST operators, $\lambda^\alpha d_\alpha$ and $\overline{\lambda}^{\overline{\alpha}}\overline{d}_{\overline{\alpha}}$, must be holomorphic and antiholomorphic. To check that this is implied by (3.10), note that the equations of motion of λ^α and $\overline{\lambda}^{\overline{\alpha}}$ coming from (3.6) are [11]

$$\begin{aligned}\overline{\nabla}\lambda^\alpha &= \frac{1}{8}R^{mnpq}(\gamma_{mn})^\alpha_\beta\lambda^\beta\overline{N}_{pq}\,,\\ \nabla\overline{\lambda}^{\overline{\alpha}} &= \frac{1}{8}R^{mnpq}(\gamma_{pq})^{\overline{\alpha}}_{\overline{\beta}}\overline{\lambda}^{\overline{\beta}}N_{mn}\,.\end{aligned} \tag{3.11}$$

So (3.10) and (3.11), together with the identity of (3.5), imply that

$$\begin{aligned}\overline{\partial}(\lambda^\alpha d_\alpha) &= \frac{1}{2}\lambda^\alpha(\gamma^{[m}F\gamma^{n]})_{\alpha\overline{\gamma}}N_{mn}\overline{L}^{\overline{\gamma}}\,,\\ \partial(\overline{\lambda}^{\overline{\alpha}}\overline{d}_{\overline{\alpha}}) &= -\frac{1}{2}\overline{\lambda}^{\overline{\alpha}}(\gamma^{[m}F\gamma^{n]})_{\gamma\overline{\alpha}}N_{mn}L^\gamma\,.\end{aligned} \tag{3.12}$$

Since $N_{mn}=\frac{1}{2}(\lambda\gamma_{mn}w)$ and $\lambda^\alpha\lambda^\beta$ is proportional to $(\lambda\gamma^{pqrst}\lambda)(\gamma_{pqrst})^{\alpha\beta}$, the right-hand side of (3.12) is proportional to $\gamma_{mn}\gamma_{pqrst}\gamma^{[m}F\gamma^{n]}$. But since $\gamma_m\gamma_{pqrst}\gamma^m=0$, one finds that

$$\gamma_{mn}\gamma_{pqrst}\gamma^{[m}F\gamma^{n]}=\gamma_{pqrst}\gamma^nF\gamma_n=\gamma_{pqrst}\gamma^n\gamma_{uvwxy}\gamma_nF^{uvwxy}=0\,. \tag{3.13}$$

So $\overline{\partial}(\lambda^\alpha d_\alpha)=\partial(\overline{\lambda}^{\overline{\alpha}}\overline{d}_{\overline{\alpha}})=0$ as desired.

When $D=4$ and $D=6$, $N=2$ worldsheet superconformal invariance implies that the left and right-moving superconformal generators must be holomorphic and antiholomorphic. In these actions, (3.10) implies that $\overline{\nabla}d_\alpha=\nabla\overline{d}_{\overline{\alpha}}=0$ since N_{mn} and $\overline{N}_{mn}$ vanish. And since the left and right-moving $N=2$ superconformal constraints in the $D=4$ and $D=6$ formalisms are constructed out of Lorentz-invariant combinations of d_α and $\overline{d}_{\overline{\alpha}}$, (3.10) implies that these constraints are holomorphic and antiholomorphic.

Up to now, the analysis of the action for the Ramond-Ramond plane wave background has been identical to the analysis of the action for the $AdS_{D/2}\times S^{D/2}$ background. The only difference between the backgrounds is that the $(2D-4)\times(2D-4)$ matrix $F^{\alpha\overline{\beta}}$ is invertible for the $AdS_{D/2}\times S^{D/2}$ background, whereas $F^{\alpha\overline{\beta}}$ is not invertible and has rank $D-2$ for the R-R plane wave background. However, as will now be shown, this difference considerably simplifies the quantum analysis of the action in the R-R plane wave background. Although the $AdS_{D/2}\times S^{D/2}$ action has only been proven to be conformally invariant at the one-loop level [14],[7] it will be possible to prove exact conformal invariance for the action in a Ramond-Ramond plane wave background.

4. Conformal invariance of the action in an R-R plane wave background

In a Ramond-Ramond plane wave background, the only nonzero components of the self-dual field strength are $F^{-j_1\cdots j_{\frac{(D-2)}{2}}}$ where j ranges over the light-cone directions $j=1$ to $(D-2)$. It is convenient to split the SO$(D-1,1)$ spinor representation labeled by α and $\overline{\alpha}$

[7]For the $AdS_5\times S^5$ action, one-loop conformal invariance has not yet been proven for the ghost contribution (3.6) to the action. For the "harmonic" version of the $AdS_3\times S^3$ action in [12], exact conformal invariance has been proven.

into SO$(D-2)$ representations labeled by (a,a') and $(\overline{a},\overline{a}')$ where $(a,a',\overline{a},\overline{a}')$ range from 1 to $(D-2)$. Using this notation,

$$\begin{array}{llll} (\gamma^-)_{ab} = \delta_{ab}\,, & (\gamma^-)_{a'b'} = 0\,, & (\gamma^+)_{ab} = 0\,, & (\gamma^+)_{a'b'} = \delta_{a'b'}\,, \\ (\gamma^-)_{\overline{a}\overline{b}} = \delta_{\overline{a}\overline{b}}\,, & (\gamma^-)_{\overline{a}'\overline{b}'} = 0\,, & (\gamma^+)_{\overline{a}\overline{b}} = 0\,, & (\gamma^+)_{\overline{a}'\overline{b}'} = \delta_{\overline{a}'\overline{b}'}\,. \end{array} \tag{4.1}$$

Since $F^{a\overline{b}'} = F^{a'\overline{b}'} = F^{a'\overline{b}} = 0$, the commutation relations of (3.4) imply that

$$\begin{aligned} &[P^-,P^j] = \mu^2 J^{+j}\,, \quad \{Q_a,Q_b\} = 2P^+\delta_{ab}\,, \quad \{Q_{\overline{a}},Q_{\overline{b}}\} = 2P^+\delta_{\overline{a}\overline{b}}\,, \\ &[P^-,Q_a] = -\delta_{ab}F^{b\overline{c}}Q_{\overline{c}}\,, \quad [P^-,Q_{\overline{a}}] = \delta_{\overline{a}\overline{b}}F^{c\overline{b}}Q_c\,, \end{aligned} \tag{4.2}$$

where $\mu^2 = F^{a\overline{b}}F^{c\overline{d}}\delta_{ac}\delta_{\overline{b}\overline{d}}$. Therefore, $[Q_{a'},Q_{\overline{a}'},J_{jk}]$ are never created from commutators of $[P_m,Q_a,Q_{\overline{a}}]$. So $[L^{a'},L^{\overline{a}'},L^{jk}]$ in (3.3) only depend on $(\theta^{a'},\overline{\theta}^{\overline{a}'})$ and are independent of $(x^m,\theta^a,\overline{\theta}^{\overline{a}})$. This implies that if $\theta^{a'}$ and $\overline{\theta}^{\overline{a}'}$ are defined to carry charge $+1$ and $d_{a'}$ and $\overline{d}_{\overline{a}'}$ are defined to carry charge -1, all terms in the action of (3.1) carry non-negative charge.

The term with zero charge in (3.1) is

$$\begin{aligned} \mathcal{S}_{(0)} = \mathcal{S}_{GS}|_{\theta^{a'}=\overline{\theta}^{\overline{a}'}=0} &+ \int d^2z \left(d_a\overline{L}^a_{(0)} + d_{a'}\overline{\partial}\theta^{a'} + \overline{d}_{\overline{a}}L^{\overline{a}}_{(0)} + \overline{d}_{\overline{a}'}\partial\overline{\theta}^{\overline{a}'} - \frac{1}{2}d_a\overline{d}_{\overline{b}}F^{a\overline{b}} \right) + \\ &+ S_{\text{comp}} + \int d^2z \left[\mathcal{L}^{\text{flat}}_{\text{ghost}} + N_{-j}\overline{L}^{-j}_{(0)} + \overline{N}_{-j}L^{-j}_{(0)} + N_{-j}N_{-k}\eta^{jk} \right], \end{aligned} \tag{4.3}$$

where $[L^m_{(0)},L^a_{(0)},L^{\overline{a}}_{(0)},L^{-j}_{(0)}] = [L^m,L^a,L^{\overline{a}},L^{-j}]|_{\theta^{a'}=\overline{\theta}^{\overline{a}'}=0}$. Also, at $\theta^{a'}=\overline{\theta}^{\overline{a}'}=0$, one can use the commutation relations of (3.4) to show that the Wess-Zumino term in (3.2) simplifies to $\frac{1}{2}(\overline{L}^a_{(0)}L^{\overline{b}}_{(0)} - L^a_{(0)}\overline{L}^{\overline{b}}_{(0)})F^{-1}_{a\overline{b}}$ where $F^{-1}_{a\overline{b}}$ is the inverse to the $(D-2)\times(D-2)$ matrix $F^{a\overline{b}}$. So one can write

$$\mathcal{S}_{GS}|_{\theta^{a'}=\overline{\theta}^{\overline{a}'}=0} = \int d^2z \left[\frac{1}{2}\eta_{mn}L^m_{(0)}L^n_{(0)} + \frac{1}{2}F^{-1}_{a\overline{b}}(\overline{L}^a_{(0)}L^{\overline{b}}_{(0)} - L^a_{(0)}\overline{L}^{\overline{b}}_{(0)}) \right]. \tag{4.4}$$

After integrating out d_a and $\overline{d}_{\overline{a}}$, one obtains the action

$$\begin{aligned} \mathcal{S}_{(0)} = \int d^2z \Bigg[&\frac{1}{2}\eta_{mn}L^m\overline{L}^n - \frac{1}{2}F^{-1}_{a\overline{b}}(3\overline{L}^a_{(0)}L^{\overline{b}}_{(0)} + L^a_{(0)}\overline{L}^{\overline{b}}_{(0)}) + d_{a'}\overline{\partial}\theta^{a'} + \overline{d}_{\overline{a}'}\partial\overline{\theta}^{\overline{a}'} + \\ &+ \mathcal{L}^{\text{flat}}_{\text{ghost}} + N_{-j}\overline{L}^{-j}_{(0)} + \overline{N}_{-j}L^{-j}_{(0)} + N_{-j}N_{-k}\eta^{jk} \Bigg] + \mathcal{S}_{\text{comp}}\,. \end{aligned} \tag{4.5}$$

As will now be shown, $\mathcal{S}_{(0)}$ is conformally invariant. This can be used to prove conformal invariance of the action of (3.1) since all terms with positive charge in (3.1) are related to $S_{(0)}$ by isometries of the action. In other words, the global isometries of the background imply that the action is constructed from the currents of (3.3) in combinations which are invariant under tangent-space Lorentz transformations. These combinations are

$$\begin{aligned} &\eta_{mn}L^m\overline{L}^n\,, \quad \int dy\,\epsilon^{IJK}(\gamma_{m\alpha\beta}L^m_I L^\alpha_J L^\beta_K + \gamma_{m\overline{\alpha}\overline{\beta}}L^m_I L^{\overline{\alpha}}_J L^{\overline{\beta}}_K)\,, \quad d_\alpha\overline{L}^\alpha\,, \quad \overline{d}_{\overline{\alpha}}L^{\overline{\alpha}}\,, \\ &d_\alpha\overline{d}_{\overline{\beta}}F^{\alpha\overline{\beta}}\,, \quad N_{mn}\overline{N}_{pq}R^{mnpq}\,, \quad \mathcal{L}^{\text{flat}}_{\text{ghost}} + \frac{1}{2}(N_{mn}\overline{L}^{mn} + \overline{N}_{mn}L^{mn})\,, \end{aligned} \tag{4.6}$$

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and only the coefficients in front of the various combinations can be adjusted without breaking the isometries. However, the coefficients are determined once one knows $S_{(0)}$, so if $S_{(0)}$ is conformally invariant, the entire action of (3.1) is conformally invariant.

To show that $S_{(0)}$ is conformally invariant, note that the first line of (4.5) has precisely the G/H coset space structure discussed in [14] where $\mathcal{H}_0 = J^{+j}$, $\mathcal{H}_1 = Q_a$, $\mathcal{H}_2 = P^m$, and $\mathcal{H}_3 = Q_{\overline{a}}$. Using the analysis of [14], one can therefore prove that the first line of (4.5) is one-loop conformally invariant. Furthermore, one can prove the exact conformal invariance of $\mathcal{S}_{(0)}$ by computing the currents

$$G^{-1}\partial G = P_m L^m_{(0)} + Q_a L^a_{(0)} + Q_{\overline{a}} L^{\overline{a}}_{(0)} + J_{-j} L^{-j}_{(0)} \tag{4.7}$$

where $G(x^m, \theta^a, \overline{\theta}^{\overline{a}}) = \exp(x^+ P^-)\exp(x^- P^+ + x^j P^j + \theta^a Q_a + \overline{\theta}^{\overline{a}} Q_{\overline{a}})$. One finds that

$$\begin{aligned}
\mathcal{S}_{(0)} = \int d^2 z \Bigg[& \frac{1}{2}\partial x^m \overline{\partial} x_m - 2F^{-1}_{a\overline{b}}\partial\theta^a \overline{\partial}\overline{\theta}^{\overline{b}} + \frac{1}{2}\partial x^+ \overline{\partial} x^+ \mu^2 x^j x^j + \partial x^+ \delta_{ab}\theta^a \overline{\partial}\theta^b + \\
& + \overline{\partial} x^+ \delta_{\overline{a}\overline{b}} \overline{\theta}^{\overline{a}} \partial \overline{\theta}^{\overline{b}} + + d_{a'}\overline{\partial}\theta^{a'} + \overline{d}_{\overline{a}'}\partial\overline{\theta}^{\overline{a}'} + \mathcal{L}^{\text{flat}}_{\text{ghost}} + \\
& + \mu^2 (x^j (\lambda^{b'} \sigma_j^{ac'} \delta_{b'c'} w_a \overline{\partial} x^+ + \overline{\lambda}^{\overline{b}'} \sigma_j^{\overline{a}\overline{c}'} \delta_{\overline{b}'\overline{c}'} \overline{w}_{\overline{a}} \partial x^+) + \\
& + (\lambda^{b'} \sigma_j^{ac'} \delta_{b'c'} w_a)(\overline{\lambda}^{\overline{b}'} \sigma^{j\overline{a}\overline{c}'} \delta_{\overline{b}'\overline{c}'} \overline{w}_{\overline{a}})) \Bigg] + \mathcal{S}_{\text{comp}} \,.
\end{aligned} \tag{4.8}$$

By separating the worldsheet variables in (4.8) into background values and quantum variables, and integrating over the quantum variables, one computes the quantum effective action. Since $[x^-, \lambda^a, w_{a'}, \overline{\lambda}^{\overline{a}}, \overline{w}_{\overline{a}'}]$ appear in the quadratic part of (4.8) but do not appear in the vertices, the variables $[x^+, w_a, \lambda^{a'}, \overline{w}_{\overline{a}}, \overline{\lambda}^{\overline{a}'}]$ can be set to their background value in the vertices of (4.8). After doing this, all quantum variables appear at most quadratically in (4.8), which means they can only give a one-loop contribution to the effective action. This one-loop contribution is easily computed to vanish where the x^+ dependence cancels after integrating over the x^j and $[\theta^a, \overline{\theta}^{\overline{a}}]$ quantum variables, and the central charge cancels for the same reason as in a flat background.

It has therefore been proven that the action of (3.1) for the superstring in an R-R plane wave background is an exact conformal field theory. It would be interesting to try to use this conformal field theory to compute scattering amplitudes. Since this conformally invariant action does not require interaction point operators, the amplitude computations might be simpler than using the light-cone gauge action. Although the action of (3.1) is more complicated than the quadratic light-cone action, there might be certain amplitude computations in which "charge conservation" of the $(\theta^{a'}, \overline{\theta}^{\overline{a}'})$ variables implies that the complicated action of (3.1) can be replaced by the simpler action of 4.8).

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